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What is claimed is:

1. A method performed by a computer for generating a geometric pattern from an image having a plurality of ridges and mesh points, the method comprising the steps of:

5 forming a partial differential equation by transferring values for positions in the image to corresponding coefficients of the partial differential equation;

10 determining simultaneous difference equations corresponding to the partial differential equation and the image mesh points;

solving the simultaneous difference equations; and

15 mapping the solutions of the simultaneous difference equations to respective positions on the image to determine features of the image.

20 2. The method of claim 1, wherein the image is a fingerprint.

25 3. The method of claim 1, wherein the image is a facial image.

4. The method of claim 1, wherein the image is a hand-palm image.

5. The method of claim 1, wherein the image is an eye iris image.

30 6. The method of claim 1, wherein the image is a texture image.

7. The method of claim 1, wherein the image is an eye retina image.

35 8. The method of claim 1, wherein the step of forming a partial differential equation comprises the steps of:

1        calculating a plurality of intrinsic properties of the image  
according to image ridge pattern;

5        mapping the plurality of intrinsic properties into  
coefficients of the partial differential equation; and

5        determining a boundary condition for the partial  
differential equation from the image to establish a relationship  
between properties of the image and the partial differential  
equation.

10       9. The method of claim 1, wherein the step of determining  
simultaneous difference equations comprises the step of replacing  
each derivative and variable items of the partial differential  
equation with respect to each mesh point in the image.

15       10. The method of claim 1, wherein the step of forming a  
partial differential equation comprises the steps of:

      determining initial conditions for the partial differential  
equation; and

      determining boundary condition for the partial differential  
equation.

20       11. The method of claim 10, wherein the step of determining  
initial conditions comprises the steps of:

25        normalizing the image to reduce variations in gray-level  
values along ridges of the image;

      estimating property values of the image; and

      mapping the estimated property values into weight  
coefficients of the partial differential equation.

30       12. The method of claim 10, wherein the step of determining  
boundary condition comprises the steps of:

      drawing a close boundary within the image; and

      setting boundary condition on the drawn boundary.

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13. The method of claim 11, wherein the step of normalizing comprises the steps of:

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determining  $M$ , mean of the gray-level in a region  $R$  by computing

$$M = \frac{1}{N} \sum_{(I,J) \in R} F(I,J) \quad (2a)$$

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where  $N$  is total number pixels in the region  $R$ ;  $F(I, J)$  is gray value of the image at point  $(I, J)$ ;

determining  $V$ , variance of the region  $R$  by computing

$$V = \frac{1}{N} \sum_{(I,J) \in R} (F(I,J) - M)^2 \quad (2b); \text{ and}$$

determining a normalized region  $R$  by computing

$$R(I,J) = m + \sqrt{v * ((F(I,J) - M)^2 / V)}, \text{ if } (I,J) > M; \quad (2c)$$

$$R(I,J) = m - \sqrt{v * ((F(I,J) - M)^2 / V)}, \text{ otherwise}$$

where  $m$  and  $v$  are the desired mean and variance values, respectively.

15 14. The method of claim 11, wherein the step of estimating property values comprises the steps of:

25

dividing a region  $R$  into blocks of size  $b*b$  as  $B(k)$  ;  
computing gradients at each pixel in  $R$  as

$$\begin{aligned} \partial x(I,J) &= (p1 * F(I-d,J) + p2 * F(I,J) + p3 * F(I+d,J)) / p, \\ \partial y(I,J) &= (p1 * F(I,J-d) + p2 * F(I,J) + p3 * F(I,J+d)) / p \end{aligned} \quad (3a)$$

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where  $\partial x(I, J)$  and  $\partial y(I, J)$  are the gradient magnitude in  $x$  and  $y$  directions at pixel  $(I, J)$  of the image respectively,  $p1$ ,  $p2$  are positive numbers,  $p2$  is negative number, and  $d$  is a constant expressed as step of the gradients  $p = p1 + p2 + p3$ ;

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estimating local orientation of each block ( $B(k)$ ) centered at pixel ( $I, J$ ) by computing

5

$$\xi_x(I, J) = \sum \sum 2 * \partial x(u, v) * \partial y(u, v), \quad (3b)$$

$$(u, v) \in B(k)$$

10

$$\xi_y(I, J) = \sum \sum (\partial \partial x(u, v) * \partial \partial y(u, v)), \quad (3c)$$

$$(u, v) \in B(k)$$

15

$$\theta(I, J) = (1/2) \text{atan}\{\xi_x(I, J) / \xi_y(I, J)\} \quad (3d)$$

Where  $\theta(I, J)$  is an estimate of the local ridge orientation at the block centered at pixel ( $I, J$ );

20

computing ridge oriental vector as

$$P = (1/n) \sum \cos(2 * \theta(I, J)) \quad (4a)$$

$$(I, J) \in R$$

$$Q = (1/n) \sum \sin(2 * \theta(I, J)) \quad (4b)$$

$$(I, J) \in R$$

25

where  $P$  and  $Q$  are first and second components of the ridge oriental vector, respectively, and  $n$  is the total number of pixels calculated at the local region  $R$ ;

for each block centered at pixel ( $I, J$ ), computing the minimal value and maximal value the pixel;

30

for each block centered at pixel ( $I, J$ ), computing a sequence of pixels  $\text{seq}(I, J)$  that take minimal and maximal value along the direction  $(a, b)$ , where  $(a, b)$  is orthogonal vector of the main oriental vector  $(p, q)$ ;

35

1       computing freq(I, J), frequency of seq(I, J) at each block  
 centered at pixel (I, J) according to the differential value  
 between connected elements in seq(I, J); and

5       estimating a local ridge frequency W by computing

$$W = \frac{1}{K} \sum \sum_{(u,v) \in \text{wnd}(I,J)} \text{freq}(u,v) \quad . \quad (5)$$

10      15. The method of claim 11, wherein the step of mapping  
 comprises the step of determining weight coefficients, A1, A2,  
 A3, A4, A5 and A6 of a partial differential equation

$$A1 \frac{\partial^2 U}{\partial X^2} + A2 \frac{\partial^2 U}{\partial X \partial Y} + A3 \frac{\partial^2 U}{\partial Y^2} + A4 \frac{\partial U}{\partial X} + A5 \frac{\partial U}{\partial Y} + A6 * U = 0 \quad (1)$$

15      by computing

$$A1 = P * P * (P * P + Q * Q) * W * W, \quad (6a)$$

$$A2 = 2 * (\sqrt{u * u - P * P * W * W}) * \sqrt{v - Q * Q * W * W}) / W, \quad (6b)$$

$$A3 = Q * Q * (P * P + Q * Q) * W * W, \quad (6c)$$

$$A4 = u * q + v, \quad (6d)$$

$$A5 = -v * p - u, \text{ and} \quad (6e)$$

$$A6 = a * (P * P + Q * Q) + b \quad (6f)$$

20      where a, b, u, v are constants.

25      16. The method of claim 12, wherein the step of setting  
 boundary condition comprises the steps of:

30      denoting B as a discrete boundary in a region R; and  
 computing

$$U \Big|_{(x,y) \in S} = B1(x,y), \quad \frac{\partial U}{\partial s} \Big|_{(x,y) \in S} = B2(x,y) \quad (7)$$

35      where S is a continuous boundary defined on the discrete  
 boundary B, and B1(x, y) and B2(x, y) are the continuous  
 function and differentiable function defined on the  
 boundary S, respectively.

1        17. The method of claim 1, wherein the step of determining  
simultaneous difference equations comprises the steps of:

5        integralizing the image to produce a group of integral  
points within a region R and an integral boundary IB;

10        discretizing the image based on the mesh points for  
numerating the partial differential equation and a boundary  
condition; and

15        transforming the discretized image by solving each mesh  
point in the image to determine the simultaneous difference  
equations.

18. The method of claim 17, wherein the step of  
integralizing comprises the steps of:

20        denoting two directions of the coordinate axes of the  
fingerprint image as X-direction and Y-direction; and

25        along the X-direction and Y-direction, computing integral  
points at a desired step length H as follow

$$X(I) = X_0 + I*H, \quad I = 0, 1, 2, \dots, W(F), \quad (8a)$$

$$Y(J) = Y_0 + J*H, \quad J = 0, 1, 2, \dots, H(F), \quad (8b)$$

20        where,  $(X_0, Y_0)$  is top left point of the image,  $W(F)$  is the  
width of the image and  $H(F)$  is the height of the image.

19. The method of claim 17, wherein the step of  
discretizing comprises the steps of:

25        computing the derivatives in the partial differential  
equation

$$A1 \frac{\partial^2 U}{\partial X^2} + A2 \frac{\partial^2 U}{\partial X \partial Y} + A3 \frac{\partial^2 U}{\partial Y^2} + A4 \frac{\partial U}{\partial X} + A5 \frac{\partial U}{\partial Y} + A6 * U = 0 \quad (1)$$

30        with respect to each inner mesh point as

$$\frac{\partial U}{\partial X} \approx [U(X+H, Y) - U(X, Y)]/H \quad (9a)$$

$$\frac{\partial U}{\partial Y} \approx [U(X, Y+H) - U(X, Y)]/H \quad (9b)$$

1       $\frac{\partial^2 U}{\partial X^2} \approx [U(X+H, Y) - 2*U(X, Y) + U(X-H, Y)]/H^2$       (9c)

5       $\frac{\partial^2 U}{\partial Y^2} \approx [U(X, Y+H) - 2*U(X, Y) + U(X, Y-H)]/(H^2)$       (9d)

5       $\frac{\partial^2 U}{\partial X \partial Y} \approx [U(X+H, Y+H) - U(X+H, Y) - U(X, Y+H) + U(X, Y)]/(H^2)$       (9e)

where  $(X, Y)$  is inner mesh point in region  $R$ ,  $IMP(R)$ ;

10     discretizing the boundary condition

10      $U \Big|_{(x,y) \in S} = B1(x,y), \quad \frac{\partial U}{\partial s} \Big|_{(x,y) \in S} = B2(x,y); \text{ and}$       (7)

15     combining the numerical derivatives of the partial differential equation and numerical boundary condition.

20. The method of claim 19, wherein the step of discretizing the boundary condition comprises the steps of:

20     replacing the continuous function  $U(X, Y)$  in the boundary condition with discrete function  $F(I, J)$ , wherein  $(I, J)$  is inner mesh point of a region in the image;

25     replacing the continuous function  $B1(x, y)$  in the boundary condition with a numerical function according to

25      $D1(X, Y) = f1*F(X, Y) + f2, \quad (X, Y) \in IB$       (10)

30     where  $f1$  and  $f2$  are constants that are predetermined according to brightness and contrast of the image, and  $F(X, Y)$  is the gray value at point  $(X, Y)$  on the integral boundary  $IB$ ; and

35     replacing the continuous function  $B2(x, y)$  in the boundary condition with a numerical function according to

35      $D2(X, Y) = f1*[F(X1, Y1) - F(X, Y)]/h, \quad (X, Y) \in IB$       (11)

h=sqrt((X1-X)\*(X1-X)+(Y1-Y)\*(Y1-Y));

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where  $(X_1, Y_1)$  is an integral point on IB selected as the next adjacent point along the boundary line IB.

5

21. The method of claim 1, wherein the step of mapping the solutions comprises the steps of:

for each region R, computing minimum value  $\min (R)$  and maximum value  $\max (R)$  in the solution;

computing ratio value  $r (R) = 255 / (\max (R) - \min (R))$ ;

10 for each point  $(I, J)$  in the region R, assigning  $v(I, J) * 3$  as the solution at point  $(I, J)$ ;

computing  $W(I, J) = r(R) * (v(I, J) - \min(R))$  as the mapping of  $w(I, J)$  into gray level byte at the position  $(I, J)$ ; and

enhancing the region R is by placing the value  $W(I, J)$  at position  $(I, J)$ .

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22. A method performed by a computer for extracting features from an image, the method comprising the steps of:

establishing a mathematical model according to regional conditions in the image;

converting the mathematical model into numerical equations; solving the numerical equations; and

transferring the solutions of the numerical equations to respective regions of the image.

25

23. The method of claim 22, wherein the step of establishing a mathematical model comprises the steps of:

forming a partial differential equation;

calculating a plurality of intrinsic properties of the image according to image ridge pattern;

30 mapping the plurality of intrinsic properties into coefficients of the partial differential equation; and

determining a boundary condition for the partial differential equation from the image to establish a relationship

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1        between properties of the image and the partial differential  
 equation.

5        24. The method of claim 22, wherein the step of  
 establishing a mathematical model comprises the steps of:

      forming a partial differential equation;

      determining initial conditions for the partial differential  
 equation; and

10      determining boundary condition for the partial differential  
 equation.

15      25. The method of claim 24, wherein the step of determining  
 initial conditions comprises the steps of:

      normalizing the image to reduce variations in gray-level  
 values along ridges of the image;

      estimating property values the image; and

      mapping the estimated property values into weight  
 coefficients of the partial differential equation.

20      26. The method of claim 24, wherein the step of determining  
 boundary condition comprises the steps of:

      drawing a close boundary within the image; and

      setting boundary condition on the drawn boundary.

25      27. The method of claim 25, wherein the step of normalizing  
 comprises the steps of:

      determining  $M$ , mean of the gray-level in a region  $R$  by  
 computing

30      
$$M = \frac{1}{N} \sum \sum F(I, J) \quad (2a)$$
  

$$(I, J) \in R$$

      where  $N$  is total number pixels in the region  $R$ ;  $F(I, J)$  is  
 gray value of the image at point  $(I, J)$ ;

35      determining  $V$ , variance of the region  $R$  by computing

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$$V = \frac{1}{N} \sum \sum (F(I,J) - M(F)) * (F(I,J) - M(F)) \quad (2b); \text{ and} \\ (I,J) \in R$$

5 determining a normalized region R by computing

$$R(I,J) = m + \sqrt{(v * (F(I,J) - M) * (F(I,J) - M)) / V}, \text{ if } (I,J) > M; \quad (2c) \\ R(I,J) = m - \sqrt{(v * (F(I,J) - M) * (F(I,J) - M)) / V}, \text{ otherwise}$$

10 where m and v are the desired mean and variance values, respectively.

28. The method of claim 25, wherein the step of estimating property values comprises the steps of:

15 dividing a region R into blocks of size b\*b as B(k);  
computing gradients at each pixel in R as

$$\partial x(I,J) = (p1 * F(I-d,J) + p2 * F(I,J) + p3 * F(I+d,J)) / p, \quad (3a) \\ \partial y(I,J) = (p1 * F(I,J-d) + p2 * F(I,J) + p3 * F(I,J+d)) / p$$

20 where  $\partial x$  (I, J) and  $\partial y$  (I, J) are the gradient magnitude in x and y directions at pixel (I, J) of the image respectively, p1, p2 are positive numbers, p2 is negative number, and d is a constant expressed as step of the gradients  $p = p1 + p2 + p3$ ;

25

estimating local orientation of each block (B(k)) centered at pixel (I, J) by computing

$$\xi_x(I,J) = \sum \sum 2 * \partial x(u,v) * \partial y(u,v), \quad (3b) \\ (u,v) \in B(k)$$

30

$$\zeta_y(I,J) = \sum \sum (\partial \partial x(u,v) * \partial \partial y(u,v)), \quad (3c) \\ (u,v) \in B(k)$$

$$\theta(I,J) = (1/2) \text{atan} \{ \xi_x(I,J) / \zeta_y(I,J) \} \quad (3d)$$

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where  $\theta(I, J)$  is an estimate of the local ridge orientation at the block centered at pixel  $(I, J)$ ;

5

computing ridge oriental vector as

$$P = \frac{1}{n} \sum_{(I, J) \in R} \cos(2 * \theta(I, J)) \quad (4a)$$

$$Q = \frac{1}{n} \sum_{(I, J) \in R} \sin(2 * \theta(I, J)) \quad (4b)$$

10

where  $P$  and  $Q$  are first and second components of the ridge oriental vector, respectively, and  $n$  is the total number of pixels calculated at the local region  $R$ ;

15

for each block centered at pixel  $(I, J)$ , computing the minimal value and maximal value the pixel;

20

for each block centered at pixel  $(I, J)$ , computing a sequence of pixels  $seq(I, J)$  that take minimal and maximal value along the direction  $(a, b)$ , where  $(a, b)$  is orthogonal vector of the main oriental vector  $(p, q)$ ;

computing  $freq(I, J)$ , frequency of  $seq(I, J)$  at each block centered at pixel  $(I, J)$  according to the differential value between connected elements in  $seq(I, J)$ ; and

25

estimating a local ridge frequency  $W$  by computing

$$W = \frac{1}{K} \sum_{(u, v) \in wnd(I, J)} freq(u, v) \quad (5)$$

30

29. The method of claim 25, wherein the step of mapping comprises the step of determining weight coefficients,  $A1, A2, A3, A4, A5$  and  $A6$  of a partial differential equation

$$A1 \frac{\partial^2 U}{\partial X^2} + A2 \frac{\partial^2 U}{\partial X \partial Y} + A3 \frac{\partial^2 U}{\partial Y^2} + A4 \frac{\partial U}{\partial X} + A5 \frac{\partial U}{\partial Y} + A6 * U = 0 \quad (1)$$

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by computing

$$A1=P*P*(P*P+Q*Q)*W*W, \quad (6a)$$

$$A2=2*(\sqrt{u*u-P*P*W*W}*\sqrt{v-Q*Q*W*W})/W, \quad (6b)$$

5

$$A3=Q*Q(P*P+Q*Q)*W*W, \quad (6c)$$

$$A4=u*q+v, \quad (6d)$$

$$A5=-v*p-u, \text{ and} \quad (6e)$$

$$A6=a*(P*P+Q*Q)+b \quad (6f)$$

where a, b, u, v are constants.

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30. The method of claim 26, wherein the step of setting boundary condition comprises the steps of:

denoting B as a discrete boundary in a region R; and computing

$$U \Big|_{(x,y) \in S} = B1(x,y), \quad \frac{\partial U}{\partial s} \Big|_{(x,y) \in S} = B2(x,y) \quad (7)$$

where S is a continuous boundary defined on the discrete boundary B, and B1(x, y) and B2(x, y) are the continuous function and differentiable function defined on the boundary S, respectively.

15

31. The method of claim 22, wherein the step of converting the mathematical model comprises the steps of:

determining simultaneous difference equations;

25 integralizing the image to produce a group of integral points within a region R and an integral boundary IB;

discretizing the image based on mesh points for numerating the partial differential equation and a boundary condition; and

30 transforming the discretized image by solving each mesh point in the image to determine the simultaneous difference equations.

20

32. The method of claim 31, wherein the step of integralizing comprises the steps of:

35

1       denoting two directions of the coordinate axes of the  
fingerprint image as X-direction and Y-direction; and

5       along the X-direction and Y-direction, computing integral  
points at a desired step length H as follow

$$X(I) = X_0 + I*H, \quad I = 0, 1, 2, \dots, W(F), \quad (8a)$$

$$Y(J) = Y_0 + J*H, \quad J = 0, 1, 2, \dots, H(F), \quad (8b)$$

where,  $(X_0, Y_0)$  is top left point of the image,  $W(F)$  is the width of the image and  $H(F)$  is the height of the image.

10      33. The method of claim 31, wherein the step of  
discretizing comprises the steps of:

computing the derivatives in the partial differential  
equation

$$A1 \frac{\partial^2 U}{\partial X^2} + A2 \frac{\partial^2 U}{\partial X \partial Y} + A3 \frac{\partial^2 U}{\partial Y^2} + A4 \frac{\partial U}{\partial X} + A5 \frac{\partial U}{\partial Y} + A6 * U = 0 \quad (1)$$

with respect to each inner mesh point as

$$\frac{\partial U}{\partial X} \approx [U(X+H, Y) - U(X, Y)]/H \quad (9a)$$

$$\frac{\partial U}{\partial Y} \approx [U(X, Y+H) - U(X, Y)]/H \quad (9b)$$

$$\frac{\partial^2 U}{\partial X^2} \approx [U(X+H, Y) - 2*U(X, Y) + U(X-H, Y)]/H^2 \quad (9c)$$

$$\frac{\partial^2 U}{\partial Y^2} \approx [U(X, Y+H) - 2*U(X, Y) + U(X, Y-H)]/H^2 \quad (9d)$$

$$\frac{\partial^2 U}{\partial X \partial Y} \approx [U(X+H, Y+H) - U(X+H, Y) - U(X, Y+H) + U(X, Y)]/H^2 \quad (9e)$$

where  $(X, Y)$  is inner mesh point in region R,  $IMP(R)$ ;

30      discretizing the boundary condition

$$U \Big|_{(x,y) \in S} = B1(x,y), \quad \frac{\partial U}{\partial s} \Big|_{(x,y) \in S} = B2(x,y) ; \text{ and} \quad (7)$$

1        combining the numerical derivatives of the partial  
 differential equation and numerical boundary condition.

5        34. The method of claim 33, wherein the step of  
 discretizing the boundary condition comprises the steps of:

replacing the continuous function  $U(X, Y)$  in the boundary condition with discrete function  $F(I, J)$ , wherein  $(I, J)$  is inner mesh point of a region in the image;

10      replacing the continuous function  $B1(x, y)$  in the boundary condition with a numerical function according to

$$D1(X, Y) = f1 * F(X, Y) + f2, \quad (X, Y) \in IB \quad (10)$$

15      where  $f1$  and  $f2$  are constants that are predetermined according to brightness and contrast of the image, and  $F(X, Y)$  is the gray value at point  $(X, Y)$  on the integral boundary  $IB$ ; and

replacing the continuous function  $B2(x, y)$  in the boundary condition with a numerical function according to

$$D2(X, Y) = f1 * [F(X1, Y1) - F(X, Y)] / h, \quad (X, Y) \in IB \quad (11)$$

$$h = \sqrt{(X1 - X)^2 + (Y1 - Y)^2};$$

20      where  $(X1, Y1)$  is an integral point on  $IB$  selected as the next adjacent point along the boundary line  $IB$ .

25      35. The method of claim 22, wherein the step of transferring the solutions comprises the steps of:

for each region  $R$  in the image, computing minimum value  $\min(R)$  and maximum value  $\max(R)$  in the solution;

30      computing ratio value  $r(R) = 255 / (\max(R) - \min(R))$ ;

for each point  $(I, J)$  in the region  $R$ , assigning  $v(I, J) * 3$  as the solution at point  $(I, J)$ ;

computing  $W(I, J) = r(R) * (v(I, J) - \min(R))$  as the mapping of  $w(I, J)$  into gray level byte at the position  $(I, J)$ ; and

1        enhancing the region R is by placing the value  $W(I, J)$  at  
position  $(I, J)$ .

5        36. The method of claim 22, wherein the image is one or  
more of a fingerprint image, a facial image, a hand-palm image,  
an eye iris, an eye retina, and a texture image.

10        37. A digital signal processor (DSP) having stored thereon  
a set of instructions including instructions for generating  
geometric pattern from an image having a plurality of ridges and  
mesh points, when executed, the instructions cause the DSP to  
perform the steps of:

15        forming a partial differential equation by transferring  
values for positions in the image to corresponding coefficients  
of the partial differential equation;

20        determining simultaneous difference equations corresponding  
to the partial differential equation and the image mesh points;

25        solving the simultaneous difference equations; and

30        mapping the solutions of the simultaneous difference  
equations to respective positions on the image to determine  
features of the image.

35        38. The DSP of claim 37, wherein the step of forming a  
partial differential equation comprises the steps of:

40        determining initial conditions for the partial differential  
equation; and

45        determining boundary condition for the partial differential  
equation.

50        39. The DSP of claim 38, wherein the step of determining  
initial conditions comprises the steps of:

55        normalizing the image to reduce variations in gray-level  
values along ridges of the image;

60        estimating property values the image; and

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mapping the estimated property values into weight coefficients of the partial differential equation.

5

40. The DSP of claim 38, wherein the step of determining boundary condition comprises the steps of:

drawing a close boundary within the image; and  
setting boundary condition on the drawn boundary.

10

41. The DSP of claim 39, wherein the step of normalizing comprises the steps of:

determining  $M$ , mean of the gray-level in a region  $R$  by computing

$$M = \frac{1}{N} \sum \sum_{(I,J) \in R} F(I,J) \quad (2a)$$

where  $N$  is total number pixels in the region  $R$ ;  $F(I, J)$  is gray value of the image at point  $(I, J)$ ;  
determining  $V$ , variance of the region  $R$  by computing

$$V = \frac{1}{N} \sum \sum_{(I,J) \in R} (F(I,J) - M)^2 \quad (2b); \text{ and}$$

determining a normalized region  $R$  by computing

$$R(I,J) = m + \sqrt{v * ((F(I,J) - M)^2 / V)}, \text{ if } (I,J) > M; \quad (2c)$$

$$R(I,J) = m - \sqrt{v * ((F(I,J) - M)^2 / V)}, \text{ otherwise}$$

where  $m$  and  $v$  are the desired mean and variance values, respectively.

30

42. The DSP of claim 39, wherein the step of estimating property values comprises the steps of:

dividing a region  $R$  into blocks of size  $b*b$  as  $B(k)$ ;  
computing gradients at each pixel in  $R$  as

35

$$\begin{aligned} \partial x(I,J) &= (p1*F(I-d,J) + p2*F(I,J) + p3*(F(I+d,J))/p, \\ \partial y(I,J) &= (p1*F(I,J-d) + p2*F(I,J) + p3*(F(I,J+d))/p \end{aligned} \quad (3a)$$

5 where  $\partial x (I, J)$  and  $\partial y (I, J)$  are the gradient magnitude in x and y directions at pixel (I, J) of the image respectively, p1, p2 are positive numbers, p2 is negative number, and d is a constant expressed as step of the gradients  $p = p1+p2+p3$ ;

10 estimating local orientation of each block ( $B(k)$ ) centered  
at pixel ( $I, J$ ) by computing

$$\xi x(I,J) = \sum \sum 2 * \partial x(u,v) * \partial y(u,v), \quad (3b)$$

$$\zeta y(I,J) = \sum_{(u,v) \in B(k)} (\partial \partial x(u,v))^* \partial \partial y(u,v)), \quad (3c)$$

$$\theta(I,J) = (1/2) \operatorname{atan} \left\{ \frac{\xi_x(I,J)}{\zeta_y(I,J)} \right\} \quad (3d)$$

Where  $\theta(I, J)$  is an estimate of the local ridge orientation at the block centered at pixel  $(I, J)$ ;

computing ridge oriental vector as

$$P = (1/n) \sum \cos(2 * \theta(I, J)) \quad (4a)$$

$$Q = \frac{1}{n} \sum_{(I,J) \in R} \sin(2\theta(I,J)) \quad (4b)$$

Where  $P$  and  $Q$  are first and second components of the ridge oriental vector, respectively, and  $n$  is the total number of pixels calculated at the local region  $R$ ;

for each block centered at pixel (I, J), computing the minimal value and maximal value the pixel;

1 for each block centered at pixel (I, J), computing a  
 sequence of pixels seq(I, J) that take minimal and maximal value  
 along the direction (a, b), where (a, b) is orthogonal vector of  
 5 the main oriental vector (p, q);

5 computing freq(I, J), frequency of seq(I, J) at each block  
 centered at pixel (I, J) according to the differential value  
 between connected elements in seq(I, J); and

10 estimating a local ridge frequency W by computing

$$W = \frac{1}{K} \sum_{(u,v) \in \text{wnd}(I,J)} \sum \text{freq}(u,v) \quad (5)$$

15 43. The DSP of claim 39, wherein the step of mapping  
 comprises the step of determining weight coefficients, A1, A2,  
 A3, A4, A5 and A6 of a partial differential equation

$$A1 \frac{\partial^2 U}{\partial X^2} + A2 \frac{\partial^2 U}{\partial X \partial Y} + A3 \frac{\partial^2 U}{\partial Y^2} + A4 \frac{\partial U}{\partial X} + A5 \frac{\partial U}{\partial Y} + A6 * U = 0 \quad (1)$$

20 by computing

$$A1 = P * P * (P * P + Q * Q) * W * W, \quad (6a)$$

$$A2 = 2 * (\sqrt{u * u - P * P * W * W}) * \sqrt{v - Q * Q * W * W}) / W, \quad (6b)$$

$$A3 = Q * Q * (P * P + Q * Q) * W * W, \quad (6c)$$

$$A4 = u * q + v, \quad (6d)$$

$$A5 = -v * p - u, \text{ and} \quad (6e)$$

$$A6 = a * (P * P + Q * Q) + b \quad (6f)$$

25 where a, b, u, v are constants.

30 44. The DSP of claim 40, wherein the step of setting  
 boundary condition comprises the steps of:

denoting B as a discrete boundary in a region R; and  
 computing

$$1 \quad U \Big|_{(x,y) \in S} = B1(x,y), \quad \frac{\partial U}{\partial s} \Big|_{(x,y) \in S} = B2(x,y) \quad (7)$$

5 where  $S$  is a continuous boundary defined on the discrete boundary  $B$ , and  $B1(x, y)$  and  $B2(x, y)$  are the continuous function and differentiable function defined on the boundary  $S$ , respectively.

10 45. The DSP of claim 37, wherein the step of determining simultaneous difference equations comprises the steps of:

15 integralizing the image to produce a group of integral points within a region  $R$  and an integral boundary  $IB$ ;

20 discretizing the image based on the mesh points for numerating the partial differential equation and a boundary condition; and

25 transforming the discretized image by solving each mesh point in the image to determine the simultaneous difference equations.

30 46. The DSP of claim 45, wherein the step of integralizing comprises the steps of:

35 denoting two directions of the coordinate axes of the fingerprint image as X-direction and Y-direction; and

40 along the X-direction and Y-direction, computing integral points at a desired step length  $H$  as follow

$$25 \quad X(I) = X_0 + I*H, \quad I = 0, 1, 2, \dots, W(F), \quad (8a)$$

$$Y(J) = Y_0 + J*H, \quad J = 0, 1, 2, \dots, H(F), \quad (8b)$$

45 where,  $(X_0, Y_0)$  is top left point of the image,  $W(F)$  is the width of the image and  $H(F)$  is the height of the image.

50 47. The DSP of claim 45, wherein the step of discretizing comprises the steps of:

55 computing the derivatives in the partial differential equation

$$1 \quad A1 \frac{\partial^2 U}{\partial X^2} + A2 \frac{\partial^2 U}{\partial X \partial Y} + A3 \frac{\partial^2 U}{\partial Y^2} + A4 \frac{\partial U}{\partial X} + A5 \frac{\partial U}{\partial Y} + A6 * U = 0 \quad (1)$$

5 with respect to each inner mesh point as

$$\frac{\partial U}{\partial X} \approx [U(X+H, Y) - U(X, Y)]/H \quad (9a)$$

$$\frac{\partial U}{\partial Y} \approx [U(X, Y+H) - U(X, Y)]/H \quad (9b)$$

$$\frac{\partial^2 U}{\partial X^2} \approx [U(X+H, Y) - 2*U(X, Y) + U(X-H, Y)]/H^2 \quad (9c)$$

$$\frac{\partial^2 U}{\partial Y^2} \approx [U(X, Y+H) - 2*U(X, Y) + U(X, Y-H)]/(H^2) \quad (9d)$$

$$\frac{\partial^2 U}{\partial X \partial Y} \approx [U(X+H, Y+H) - U(X+H, Y) - U(X, Y+H) + U(X, Y)]/(H^2) \quad (9e)$$

10 where  $(X, Y)$  is inner mesh point in region R,  $IMP(R)$ ;

15 discretizing the boundary condition

$$20 \quad U \Big|_{(x,y) \in S} = B1(x,y), \quad \frac{\partial U}{\partial s} \Big|_{(x,y) \in S} = B2(x,y) \quad ; \text{ and} \quad (7)$$

25 combining the numerical derivatives of the partial differential equation and numerical boundary condition.

48. The DSP of claim 47, wherein the step of discretizing the boundary condition comprises the steps of:

25 replacing the continuous function  $U(X, Y)$  in the boundary condition with discrete function  $F(I, J)$ , wherein  $(I, J)$  is inner mesh point of a region in the image;

30 replacing the continuous function  $B1(x, y)$  in the boundary condition with a numerical function according to

$$35 \quad D1(X, Y) = f1 * F(X, Y) + f2, \quad (X, Y) \in IB \quad (10)$$

where  $f1$  and  $f2$  are constants that are predetermined according to brightness and contrast of the image, and  $F(X,$

1

Y) is the gray value at point (X, Y) on the integral boundary IB; and

5 replacing the continuous function  $B2(x, y)$  in the boundary condition with a numerical function according to

$$D2(X,Y)=f1*[F(X1,Y1)-F(X,Y)]/h, \quad (X,Y) \in IB \quad (11)$$

$$h=\sqrt{(X1-X)^2+(Y1-Y)^2};$$

10 where (X1, Y1) is an integral point on IB selected as the next adjacent point along the boundary line IB.

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15 49. The DSP of claim 37, wherein the step of mapping the solutions comprises the steps of:

for each region R, computing minimum value  $\min(R)$  and maximum value  $\max(R)$  in the solution;

computing ratio value  $r(R) = 255 / (\max(R) - \min(R))$ ;

for each point (I, J) in the region R, assigning  $v(I, J) * 3$  as the solution at point (I, J);

computing  $W(I, J) = r(R) * (v(I, J) - \min(R))$  as the mapping of  $w(I, J)$  into gray level byte at the position (I, J); and

enhancing the region R is by placing the value  $W(I, J)$  at position (I, J).

25 50. The DSP of claim 37, wherein the image is one or more of a finger print, a facial image, a hand-palm image, an eye iris, an eye retina, and a texture image.

30 51. A method performed by a computer for biometric image processing, the DSP comprising the steps of:

establishing numerical relationship between visual appearance of the biometric image; and

approximating solutions of a partial differential equation with a boundary condition according to the established numerical relationship to determine features of the biometric image.

35

1        52. The method of claim 51, wherein the step of establishing numerical relationship comprises the steps of:

      forming a partial differential equation;

5        calculating a plurality of intrinsic properties of the image according to image ridge pattern;

      mapping the plurality of intrinsic properties into coefficients of the partial differential equation; and

10        determining a boundary condition for the partial differential equation from the image to establish a relationship between properties of the image and the partial differential equation.

15        53. The method of claim 52, wherein the step of forming a partial differential equation comprises the steps of:

      determining initial conditions for the partial differential equation; and

      determining boundary condition for the partial differential equation.

20        54. The method of claim 53, wherein the step of determining initial conditions comprises the steps of:

      normalizing the image to reduce variations in gray-level values along ridges of the image;

25        estimating property values the image; and

      mapping the estimated property values into weight coefficients of the partial differential equation.

30        55. The method of claim 53, wherein the step of determining boundary condition comprises the steps of:

      drawing a close boundary within the image; and

      setting boundary condition on the drawn boundary.

35        56. The method of claim 51, wherein the step of approximating solutions comprises the steps of:

1        integralizing the image to produce a group of integral  
points within a region R and an integral boundary IB;

5        discretizing the image based on mesh points for numerating  
a partial differential equation and a boundary condition;

transforming the discretized image by solving each mesh  
point in the image to determine the simultaneous difference  
equations;

10      replacing the continuous function  $U(X, Y)$  in the boundary  
condition with discrete function  $F(I, J)$ , wherein  $(I, J)$  is inner  
mesh point of a region in the image;

replacing the continuous function  $B1(x, y)$  in the boundary  
condition with a numerical function according to

$$D1(X, Y) = f1 * F(X, Y) + f2, \quad (X, Y) \in IB \quad (10)$$

15      where  $f1$  and  $f2$  are constants that are predetermined  
according to brightness and contrast of the image, and  $F(X, Y)$   
Y is the gray value at point  $(X, Y)$  on the integral  
boundary IB; and

20      replacing the continuous function  $B2(x, y)$  in the boundary  
condition with a numerical function according to

$$D2(X, Y) = f1 * [F(X1, Y1) - F(X, Y)] / h, \quad (X, Y) \in IB \quad (11)$$

$$h = \sqrt{(X1 - X)^2 + (Y1 - Y)^2};$$

25      where  $(X1, Y1)$  is an integral point on IB selected as the  
next adjacent point along the boundary line IB.

57. The method of claim 56, wherein the step of  
integralizing comprises the steps of:

30      denoting two directions of the coordinate axes of the  
fingerprint image as X-direction and Y-direction; and

along the X-direction and Y-direction, computing integral  
points at a desired step length H as follow

$$X(I) = X0 + I * H, \quad I = 0, 1, 2, \dots, W(F), \quad (8a)$$

$$Y(J) = Y0 + J * H, \quad J = 0, 1, 2, \dots, H(F), \quad (8b)$$

1 where,  $(X_0, Y_0)$  is top left point of the image,  $W(F)$  is the width of the image and  $H(F)$  is the height of the image.

5 58. The method of claim 56, wherein the step of discretizing comprises the steps of:

computing the derivatives in the partial differential equation

10 
$$A1 \frac{\partial^2 U}{\partial X^2} + A2 \frac{\partial^2 U}{\partial X \partial Y} + A3 \frac{\partial^2 U}{\partial Y^2} + A4 \frac{\partial U}{\partial X} + A5 \frac{\partial U}{\partial Y} + A6 * U = 0 \quad (1)$$

with respect to each inner mesh point as

15 
$$\frac{\partial U}{\partial X} \approx [U(X+H, Y) - U(X, Y)]/H \quad (9a)$$

$$\frac{\partial U}{\partial Y} \approx [U(X, Y+H) - U(X, Y)]/H \quad (9b)$$

$$\frac{\partial^2 U}{\partial X^2} \approx [U(X+H, Y) - 2*U(X, Y) + U(X-H, Y)]/H^2 \quad (9c)$$

$$\frac{\partial^2 U}{\partial Y^2} \approx [U(X, Y+H) - 2*U(X, Y) + U(X, Y-H)]/H^2 \quad (9d)$$

$$\frac{\partial^2 U}{\partial X \partial Y} \approx [U(X+H, Y+H) - U(X+H, Y) - U(X, Y+H) + U(X, Y)]/H^2 \quad (9e)$$

where  $(X, Y)$  is inner mesh point in region  $R$ ,  $IMP(R)$ ;

25 discretizing the boundary condition

$$U \Big|_{(x,y) \in S} = B1(x,y), \quad \frac{\partial U}{\partial s} \Big|_{(x,y) \in S} = B2(x,y) \quad ; \text{ and} \quad (7)$$

30 combining the numerical derivatives of the partial differential equation and numerical boundary condition.

59. The method of claim 56, wherein the step of discretizing the boundary condition comprises the steps of:

replacing the continuous function  $U(X, Y)$  in the boundary condition with discrete function  $F(I, J)$ , wherein  $(I, J)$  is inner mesh point of a region in the image;

replacing the continuous function  $B1(x, y)$  in the boundary condition with a numerical function according to

$$D1(X, Y) = f1 * F(X, Y) + f2, \quad (X, Y) \in IB \quad (10)$$

where  $f_1$  and  $f_2$  are constants that are predetermined according to brightness and contrast of the image, and  $F(X, Y)$  is the gray value at point  $(X, Y)$  on the integral boundary IB; and

replacing the continuous function  $B2(x, y)$  in the boundary condition with a numerical function according to

$$D2(X, Y) = f1 * [F(X1, Y1) - F(X, Y)] / h, \quad (X, Y) \in IB$$

$$h = \sqrt{(X1 - X)^2 + (Y1 - Y)^2}; \quad (11)$$

where  $(X_1, Y_1)$  is an integral point on  $IB$  selected as the next adjacent point along the boundary line  $IB$ .

60. The method of claim 56, further comprising the step of mapping the solutions of the partial differential equation to respective positions on the image to determine features of the image.

61. The method of claim 60, wherein the step of mapping the solutions comprises the steps of:

for each region  $R$ , computing minimum value  $\min(R)$  and maximum value  $\max(R)$  in the solution;

computing ratio value  $r(R) = 255 / (\max(R) - \min(R))$ ;

for each point  $(I, J)$  in the region  $R$ , assigning  $v(I, J) * 3$  as the solution at point  $(I, J)$ ;

computing  $W(I, J) = r(R) * (v(I, J) - \min(R))$  as the mapping of  $w(I, J)$  into gray level byte at the position  $(I, J)$ ; and

1

enhancing the region  $R$  is by placing the value  $W(I, J)$  at position  $(I, J)$ .

5

62. The method of claim 51, wherein the image is one or more of a finger print, a facial image, a hand-palm image, an eye iris, an eye retina, and a texture image.

10

100 99 98 97 96 95 94 93 92 91 90 89 88 87 86 85 84 83 82 81 80 79 78 77 76 75 74 73 72 71 70 69 68 67 66 65 64 63 62 61 60 59 58 57 56 55 54 53 52 51 50 49 48 47 46 45 44 43 42 41 40 39 38 37 36 35 34 33 32 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

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